DEFINE-MATTER

Model Manual | Version 1.1

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Website: www.define-model.org

May 2026

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1. Overview

The DEFINE-MATTER model is a simplified model of the DEFINE (Dynamic Ecosystem-FINance-Economy) modelling framework developed by Dafermos et al. (2017), Dafermos et al. (2018), Dafermos and Nikolaidi (2019), Dafermos and Nikolaidi (2021) and Dafermos and Nikolaidi (2022).[1] DEFINE-MATTER shows how economic activity leads to the extraction of matter and the generation of waste. When production takes place, a specific amount of matter is necessary. This matter can be either extracted from the environment or come from recycling. When matter is extracted, the material reserves (i.e. those volumes of matter expected to be produced economically using the existing technology) tend to decline.

Non-fossil energy (that relies on carbon) is also necessary for the production process. Once production has taken place, the material content of this energy is released to the environment in the form of carbon emissions. The production process generates consumption and investment goods that are accumulated in the socio-economic system. The material content of these goods is called 'socio-economic material stock'. A part of this socio-economic material stock is demolished/discarded every year. Through waste management, a proportion of demolished/discarded socio-economic stock is recycled. The rest of it becomes waste that is discarded to the environment. Part of this waste is hazardous and can have negative effects both on the environment and the health of the population.

Table 1 depicts the physical stock-flow matrix for matter. The opening stock captures the volume of material stocks at the beginning of the period. During each period, matter is transformed or is transferred from one material stock to another. However, in line with the Law of Conservation of Mass, matter cannot disappear or be destroyed.[2] This means that the closing stock of matter that is shown in the last row should be the same as the opening stock of matter.

Symbols with a plus sign denote inputs into a specific stock. For example, recycling (REC_t) allows matter to return to the socio-economic material stock ($SEMS_t$) and the conversion of material resources into reserves (CON_{Mt}) increases material reserves (REV_{Mt}). Symbols with a minus sign denote outflows from a stock. For instance, material extraction (M_t) reduces material reserves (REV_{Mt}) and carbon released during fossil combustion (CON_{Ft}) reduces the carbon content of fossil resources (RES_{CFt}).

[1] More information about the DEFINE modeling framework is available here.
[2] For the use of the material balance in material flow accounting, see Fischer-Kowalski et al. (2011).

Table 1: Physical stock-flow matrix, matter

	Carbon content of fossil resources	**Carbon content of fossil reserves**	**Cumulative carbon emissions**	**Material resources**	**Material reserves**	**Socio-economic material stock**	**Cumulative material waste**	**Total**
Opening stock	RES_{CFt-1}	REV_{CFt-1}	$CARBON_{t-1}$	RES_{Mt-1}	REV_{Mt-1}	$SEMS_{t-1}$	$WASTE_{t-1}$	$MATTER_T$
Extracted non-carbon matter					-M_t	+M_t		0
Discarded matter						-$DISC_t$	+$DISC_t$	0
Recycled matter						+REC_t	-REC_t	0
Material resources converted into reserves				-CON_{Mt}	+CON_{Mt}			0
Fossil resources converted into reserves	-CON_{Ft}	+CON_{Ft}						0
Carbon released during fossil combustion		-C_{COMBt}	+C_{COMBt}					0
Carbon used in material goods		-C_{MATt}				+C_{MATt}		0
Closing stock	RES_{CFt}	REV_{CFt}	$CARBON_t$	RES_{Mt}	REV_{Mt}	$SEMS_t$	$WASTE_t$	$MATTER_T$

Note: The table refers to annual global stocks and flows. Matter is measured in gigatonnes. The cells highlighted in grey denote the stocks and flows that act as residuals in the accounting identities.

2. Equations

2.1. Matter, recycling and waste

The output produced, denoted by Y_t, embodies a specific amount of matter, MY_t, which is necessary for production (Eq. (1)). Material intensity (μ_t) is defined as the matter included per output produced. A specific proportion of the matter embodied in material goods (c_{mat}) is carbon due to fossil-derived materials, such as plastics and asphalt. The carbon content of material goods is denoted by C_{MAT} (Eq. (2)). As shown in Eq. (3), the matter embodied in goods comes from the extraction of metals and non-metallic minerals (M_t), the demolished/discarded socio-economic materials stock that is recycled (REC_t) and the carbon that is extracted and is used in material goods. REC_t is specified in Eq. (4); ρ_t denotes the recycling rate, which is defined as the ratio of recycled matter to the amount of demolished/discarded socio-economic material stock ($DISC_t$). Eq. (5) shows that the demolished/discarded socio-economic stock is equal to the socio-economic material stock ($SEMS_t$) times its depreciation rate, δ.

$$MY_t = \mu_t Y_t \tag{1}$$

$$C_{MATt} = c_{mat} MY_t \tag{2}$$

$$M_t = MY_t - C_{MATt} - REC_t \tag{3}$$

$$REC_t = \rho_t DISC_t \tag{4}$$

$$DISC_t = \delta SEMS_{t-1} \tag{5}$$

Eq. (6) shows that the socio-economic material stock increases as a result of the production of new goods that require extracted matter (metals and non-metallic minerals), recycled matter and extracted carbon ($M_t + REC_t + C_{MATt}$), while it decreases due to the demolition/discard of old material goods. The change in $WASTE_t$ increases with the demolished/discarded socio-economic stock and decreases with recycling (Eq.(7)).

$$SEMS_t = SEMS_{t-1} + M_t + C_{MATt} + REC_t - DISC_t \tag{6}$$

$$WASTE_t = WASTE_{t-1} + DISC_t - REC_t \tag{7}$$

2.2. Material resources and reserves

The non-carbon material stock dynamics are presented in Eqs. (8)-(10). Eq. (8) shows that the material reserves (REV_{Mt}) decline when matter is extracted (to be used in the production of goods) and increase when resources are converted into reserves. The annual conversion (CON_{Mt}) is given by Eq. (9). An exogenous conversion rate, denoted by con_M, has been assumed. Eq. (10) shows that material resources (RES_{Mt}) that reduced due to their conversion into reserves. To capture the scarcity of matter, we define the matter depletion ratio (dep_{Mt}), which is the ratio of matter that is extracted every year relative to the remaining material reserves (Eq. (11)). The higher this ratio, the greater the matter depletion problems.

$$REV_{Mt} = REV_{Mt-1} + CON_{Mt} - M_t \tag{8}$$

$$CON_{Mt} = con_M RES_{Mt-1} \tag{9}$$

$$RES_{Mt} = RES_{Mt-1} - CON_{Mt} \tag{10}$$

$$dep_{Mt} = \frac{M_t}{REV_{Mt-1}} \tag{11}$$

2.3. Carbon-related resources and reserves

The carbon-related stock dynamics are captured by Eqs. (12)-(16). Eq. (12) shows that fossil reserves (REV_{CFt}) decline when carbon is released because of fossil combustion (C_{COMBt}) and when carbon is used in material goods (C_{MATt}). They increase when fossil resources are converted into reserves (CON_{Ft}). The annual conversion of these fossil resources (CON_{Ft}) is given by Eq. (13). con_F is the exogenous conversion rate. As shown in Eq. (14), the carbon released during fossil combustion is calculated from the fossil emissions ($EMIS_{Ft}$) using the conversion rate of Gt of carbon into Gt of CO_2 (car). Eq. (15) shows that the cumulative carbon emissions increase as carbon is released during fossil combustion.[3] Eq. (16) shows the decline in fossil resources (RES_{CFt}) due to their conversion into reserves (CON_{Ft}). We define the fossil depletion ratio (dep_{CFt}) as the ratio of carbon released during fossil combustion plus the carbon used in material goods, relative to the remaining fossil reserves (Eq. (17)).

$$REV_{CFt} = REV_{CFt-1} + CON_{Ft} - C_{COMBt} - C_{MATt} \tag{12}$$

$$CON_{Ft} = con_F RES_{CFt-1} \tag{13}$$

$$C_{COMBt} = \frac{EMIS_{Ft}}{car} \tag{14}$$

$$CARBON_t = CARBON_{t-1} + C_{COMBt} \tag{15}$$

$$RES_{CFt} = RES_{CFt-1} - CON_{Ft} \tag{16}$$

$$dep_{CFt} = \frac{C_{COMBt} + C_{MATt}}{REV_{CFt-1}} \tag{17}$$

Eq. (18) is derived by combining the closing and the opening stock rows in Table 1. Since the matter ($MATTER_T$) is constant, it is netted out.[4] Eq. (18) is used as the redundant equation of the model. In the simulations, we need to check that the value of fossil resources derived via this equation (RES_{CFredt}) is the same as the value derived via Eq. (16).

$$\begin{aligned} RES_{CFredt} = {} & RES_{CFt-1} + REV_{CFt} - REV_{CFt-1} + CARBON_t - CARBON_{t-1} + RES_{Mt} - RES_{Mt-1} \\ & + REV_{Mt} - REV_{Mt-1} + SEMS_t - SEMS_{t-1} + WASTE_t - WASTE_{t-1} \end{aligned} \tag{18}$$

2.4. Ecological efficiency indicators

Green capital accumulation contributes to a lower material intensity (μ_t) and a higher recycling rate (ρ_t). We postulate that material intensity and recycling improve when the ratio of green capital (K_{Gt}) to conventional capital (K_{Ct}) rises. This is captured by Eqs. (19) and (20).$\mu_{\min}$ is the minimum potential value of material intensity. This minimum value is approached when green energy capital becomes sufficiently high compared to the conventional energy capital of each sector. $\rho_{\max}$ is the maximum potential value of the recycling rate, which is approached when $\frac{K_{Gt}}{K_{Ct}}$ becomes sufficiently high. $\mu_{\max}$ is the maximum material intensity, which is achieved when green capital is equal to zero. The use of logistic functions in Eqs. (19) and (20) allows us to take into account learning processes which play a key role in the diffusion and efficiency of new technologies.

$$\mu_t = \mu_{min} + \frac{\mu_{max} - \mu_{min}}{1 + ((K_{Gt-1}/K_{Ct-1})/c_\mu)^{k_\mu}} \tag{19}$$

$$\rho_t = \frac{1}{1 + ((K_{Gt-1}/K_{Ct-1})/c_\rho)^{k_\rho}} \tag{20}$$

[3] For simplicity, we have assumed away the carbon that is released from the degradation of fossil-derived materials.

[4] Note that $MATTER_T = RES_{CFt} + REV_{CFt} + CARBON_t + RES_{Mt} + REV_{Mt} + SEMS_t + WASTE_t$.

2.5. Auxiliary equations

Fossil emissions, output, green capital stock (K_{Gt}) and conventional capital stock (K_{Ct}) are determined exogenously (Eqs. (21)–(24)). g_{EMISF}, g_Y, g_{KG} and g_{KC} are the growth rates for fossil emissions, output, green capital stock and conventional capital stock, respectively. Eq. (24) shows that the total capital stock (K_t) is given, by definition, as the sum of green and conventional capital stock.

$$EMIS_{Ft} = EMIS_{Ft-1}(1 + g_{EMISF}) \tag{21}$$

$$Y_t = Y_{t-1}(1 + g_Y) \tag{22}$$

$$K_{Gt} = K_{Gt-1}(1 + g_{KG}) \tag{23}$$

$$K_{Ct} = K_{Ct-1}(1 + g_{KC}) \tag{24}$$

$$K_t = K_{Gt} + K_{Ct} \tag{25}$$

3. Symbols and values for variables and parameters

Table 2: Symbols and initial values for endogenous variables (baseline scenario)

Symbol	Description	Variable category	Initial value	Source/remarks
μ	Material intensity	Model-constrained	0.4948	Calculated using Eq. (1)
ρ	Recycling rate	Model-constrained	0.1031	Calculated using Eq. (4)
C_{COMB}	Carbon released during fossil combustion (Gt)	Model-constrained	10.31	Calculated using Eq. (14)
C_{MAT}	Carbon embodied in material goods (Gt)	Model-constrained	0.5291	Calculated using Eq. (2)
$CARBON$	Cumulative carbon emissions (Gt)	Free	494.6	Taken from Global Carbon Budget
CON_F	Conversion of fossil resources to reserves (Gt)	Model-constrained	45.6811	Calculated using Eq. (13)
CON_M	Conversion of material resources to reserves (Gt)	Model-constrained	2.2534	Calculated using Eq. (9)
dep_{CF}	Depletion of fossil reserves	Free	0.007	Selected from a reasonable range of values
dep_M	Depletion of material reserves	Free	0.026	Based on Jowitt et al, 2020
$DISC$	Discarded matter (Gt)	Model-constrained	57.0353	Calculated using Eq. (5)
$EMIS_F$	Fossil emissions (GtCO2)	Free	37.8068	Taken from Global Carbon Budget
g_{CARBON}	Growth rate of cumulative carbon emissions	Model-constrained	0.0213	Calculated using Eq. (15)
g_{RESCF}	Growth rate of fossil resources	Model-constrained	-0.015	Calculated using Eq. (16)
g_{RESM}	Growth rate of material resources	Model-constrained	-0.0015	Calculated using Eq. (10)
g_{REVCF}	Growth rate of fossil reserves	Model-constrained	0.0225	Calculated using Eq. (17)
g_{REVM}	Growth rate of material reserves	Model-constrained	-0.0247	Calculated using Eq. (11)
g_{SEMS}	Growth rate of socio-economic material stock	Model-constrained	-0.0035	Calculated using Eq. (6)
g_{WASTE}	Growth rate of waste	Free	0.029	Determined based on initial growth rate of output
K	Capital stock (USD trillion)	Free	320.82	Selected from a reasonable range of values
K_C	Conventional capital stock (USD trillion)	Model-constrained	304.779	Calculated using Eq. (23)
K_G	Green capital stock (USD trillion)	Free	16.041	Selected from a reasonable range of values
M	Extracted non-carbon matter (Gt)	Model-constrained	46.5009	Calculated using Eq. (3)
MY	Material content of output (Gt)	Free	52.91	Taken from Wiedenhofer et al., 2019
REC	Recycled matter (Gt)	Free	5.88	Taken from Wiedenhofer et al., 2019
RES_{CF}	Fossil resources (Gt)	Free	2999.7273	McGlade and Ekins, 2015
RES_{CFred}	Fossil resources (redundant) (Gt)	Free	2999.7273	Redundant variable
RES_M	Material resources (Gt)	Free	1500	Based on USGS, 2024
REV_{CF}	Fossil reserves (Gt)	Model-constrained	1583.2849	Calculated using Eq. (12)
REV_M	Material reserves (Gt)	Model-constrained	1744.2486	Calculated using Eq. (8)
$SEMS$	Socio-economic material stock (Gt)	Free	1184.11	Taken from Wiedenhofer et al., 2019
$WASTE$	Waste (Gt)	Model-constrained	1815.131	Calculated using Eq. (7)
Y	Output (USD trillion)	Free	106.94	Taken from World Bank data, 2023 prices

Table 3: Symbols and values for parameters (baseline scenario)

Symbol	Description	Parameter category	Value	Source/remarks
δ	Depreciation rate of capital stock	Free	0.048	Based on Penn World Table 10.0
μ_{max}	Maximum value of material intensity	Free	0.5937	Calibrated such that the model generates the baseline scenario
μ_{min}	Minimum value of material intensity	Free	0.2474	Calibrated such that the model generates the baseline scenario
c_{mat}	Share of carbon in the matter embodied in goods	Free	0.01	Selected from a reasonable range of values
c_{mu}	Green capital ratio at which material intensity takes its mean value	Model-constrained	0.2057	Calculated using Eq. (19)
c_{rho}	Green capital ratio at which the recycling rate takes its mean value	Model-constrained	8.0912e+06	Calculated using Eq. (20)
car	Conversion rate of carbon into CO2	Free	3.667	Taken from EPA
con_F	Rate of fossil resources converted to reserves	Free	0.015	Selected from a reasonable range of values
con_M	Rate of material resources converted to reserves	Free	0.0015	Selected from a reasonable range of values
g_{EMISF}	Growth rate of fossil emissions	Free	0.029	Determined based on the initial growth rate of output
g_{KC}	Growth rate of conventional capital stock	Free	0.029	Determined based on the initial growth rate of output
g_{KG}	Growth rate of green capital stock	Free	0.039	Determined based on the initial growth rate of output
g_Y	Growth rate of output	Free	0.029	Based on World Bank
k_{mu}	Steepness of material intensity improvement	Free	0.6721	Calibrated such that the model generates the baseline scenario
k_{rho}	Steepness of recycling rate improvement	Free	-0.1148	Calibrated such that the model generates the baseline scenario

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